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# MODIFIED RATIO TYPE ESTIMATOR UNDER SYSTEMATIC SAMPLING

*[1]*Dr. Sitanshu Shekhar Mishra*, [2]*Dr. Archana Panigrahi*

[1]Lecturer in Statistics, Salipur Autonomous College, Salipur, Odisha.

[2]Assistant Professor, Dept of Statistics, Ravenshaw University, Cuttack, Odisha.

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***Corresponding Author: Dr. Sitanshu Shekhar Mishra**

Lecturer in Statistics, Salipur Autonomous College, Salipur, Odisha.

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**ABSTRACT:**

In this paper we propose a modified ratio type estimator for estimating the finite population mean of the study variable in systematic sampling. The expressions of bias and mean square error (MSE) up to the first order of approximation are derived and efficiency comparison made with some existing estimators theoretically with the help of numerical example.

**KEYWORDS:** Ratio type estimator, Systematic sampling, Bias, Mean square error (M.S.E)

## 1. INTRODUCTION

Sample surveys use auxiliary data during the estimation stage to generate more efficient estimators of population parameters. Ratio, product, and regression estimators remain the most widely adopted techniques for integrating this information.

Swain (1964) proposed a ratio estimator of population mean whenever the sample is selected through systematic sampling. Shukla (1971) suggested a product estimator of population mean in systematic sampling. Following Bahl and Tuteja (1991), Singh et al. (2011) developed both ratio type exponential estimator and product type exponential estimator for estimating the population mean in systematic sampling. Tailor et al. (2013) proposed a ratio-cum-product estimator in systematic sampling for population mean using two auxiliary variables. Many researchers such as Singh (1967), Kushwaha and Singh (1989), Singh and Singh (1998), Singh and Solanki (2012), Verma and Singh (2014) and many others discussed various estimators of population mean in systematic sampling.

Let us consider a finite population $U = (U_1, U_2 .... U_N)$ of size N numbered from I to N. Assume that N = nk, where n and k are positively integers. There will be k samples (clusters) each of size n We, select a sample at random out of k samples and observe the study variable y and auxiliary variable x for each and evenly unit selected the sample. Let $y_{ij}$, $x_{ij}$ for (i=1,2…...k, j=1,2…...n) denote the value of the unit in the $i^{th}$ sample of study variable y and auxiliary variable X. Some notations are defined below:

$\overline{X} = \frac{1}{N}\sum_{i=1}^{N} x_{ij}, \overline{Y} = \frac{1}{N}\sum_{i=1}^{N} y_{ij}$

$\overline{y}_{sys} = \overline{y}_{i.} = \frac{1}{n}\sum_{j=1}^{n} y_{ij},\ \overline{x}_{sys} = \overline{x}_{i.} = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$

It is to be noted that $\overline{y}_{sys}$ and $\overline{x}_{sys}$ are unbiased estimator of population mean $\overline{Y}$ and $\overline{X}$ respectively.

Consider some estimators of the finite population mean from the literature of systematic sampling.

The simple mean estimator under systematic random sampling is given by:

$t_0 = \frac{1}{n}\sum_{j=1}^{n} y_{ij}\ (i = 1,2,3, \ldots\ldots, k)$

Its variance is given by

$Var\ (t_0) = \theta\ \overline{Y}^2\ \rho_y^*\ C_y^2$

where $\theta = \frac{N-1}{nN}$

Swain (1964) suggested the ratio estimator of population $\overline{Y}$ of the study variate based on the systematic sample as:

$$t_1 = \overline{y}_{sys}\left[\frac{\overline{X}}{\overline{x}_{xys}}\right] \tag{1.1}$$

Shukla (1971) proposed the product estimator of the finite population mean in systematic sampling as:

$$t_2 = \overline{y}_{sys}\left[\frac{\overline{x}_{xys}}{\overline{X}}\right] \tag{1.2}$$

Motivated by Singh (1967), Tailor et al. (2011) define the following ratio-cum product estimator for the population mean $\overline{Y}$ as:

$$t_5 = \overline{y}_{sy}\left[\frac{\overline{X}}{\overline{x}_{sy}}\right]\left[\frac{\overline{z}_{sy}}{\overline{Z}}\right] \tag{1.3}$$

Following Bahl and Tuteja (1991), Singh et al. (2011) utilising the known knowledge of the auxiliary variable suggested the following ratio and product type exponential estimators respectively for estimating the population mean $\overline{Y}$ in systematic sampling.

$$t_3 = \bar{y}_{sys} \exp\left[\frac{\bar{X}-\bar{x}_{sys}}{\bar{X}+\bar{x}_{sys}}\right] \quad (1.4)$$

$$t_4 = \bar{y}_{syss} \exp\left[\frac{\bar{x}_{sys}-\bar{X}}{\bar{x}_{sys}+\bar{X}}\right] \quad (1.5)$$

In the present paper we suggest a modified ratio type estimator for estimating the finite population mean of the study variable in systematic sampling. The Bias and MSE formula for the modified ratio type estimator are derived. The comparison of the proposed modified ratio type estimator with other estimators from literature are made theoretically with the help of numerical example.

## 2. Proposed Estimator

Following Swain (2014), we consider a ratio type estimator for $\bar{Y}$ in systematic sampling as:

$$t = \bar{y}_{sy}\left[\frac{\bar{X}}{\bar{x}_{sy}}\right]^{½} \quad (2.1)$$

With the help of the above estimator we propose the following modified ratio type estimator to estimate finite population mean in systematic sampling when the study variate y is positively correlated with the auxiliary variable x.

$$T = \alpha\bar{y}_{sys} + (1-\alpha)\bar{y}_{sys}\left[\frac{\bar{X}}{\bar{x}_{sy}}\right]^{½} \quad (2.2)$$

where α is a real constant to be chosen suitably.

To obtain the Bias and MSE of the proposed estimator up to 0 (1/n), we define

$$e_0 = \frac{\bar{y}_{sys}-\bar{Y}}{\bar{Y}}, e_1 = \frac{\bar{x}_{sys}-\bar{X}}{\bar{X}}$$

$E(e_0) = E(e_1) = 0$

$E(e_0^2) = \theta C_y^2 \rho_y^*$

$E(e_1^2) = \theta C_x^2 \rho_x^*$

$E(e_0e_1) = \theta K C_x^2 \sqrt{\rho_y^* \rho_x^*}$

where $C_y^2 = S_y^2/\bar{Y}^2$, $C_x^2 = S_x^2/\bar{X}^2$

$\rho_{yx} = \frac{S_{yx}}{S_y S_x}$, $K = \rho_{yx}\frac{C_y}{C_x}$

$\rho_y^* = \{1 + \rho_y (n-1)\}$

$\rho_x^* = \{1+ \rho_x (n-1) \}$

$\rho^{**} = \frac{\rho_y^*}{\rho_x^*}$

where $\rho_y$ , $\rho_x$ are intra class correlation among the pair of units for the variables x and y respectively.

$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \overline{Y})^2$

$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{k} \quad (x_{ij} - \overline{X})^2$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \overline{X})(y_{ij} - \overline{Y})$$

Be the population variance of y, population variance of x and population covariance of $y$ and $x$ respectively.

Expressing the proposed estimator T in terms of e's we have

$T= \overline{Y}\alpha(1 + e_0) + \overline{Y}(1 - \alpha)(1 + e_0) \ [1 + e_1]^{-½}$

$=\overline{Y}\alpha(1 + e_0) + \overline{Y}(1 - \alpha)(1 + e_0)(1-\frac{1}{2}e_1 + \frac{3}{8}e_1^2)$

$= \overline{Y}\left(1 + e_0 - \frac{1}{2}e_1 + \frac{3}{8}e_1^2 - \frac{1}{2}e_0e_1 + \frac{1}{2}\alpha e_1 - \frac{3}{8}\alpha e_1^2 + \frac{1}{2}\alpha e_0e_1\right)$

$B(T) = E(T) - \overline{Y}$

$= (1 - \alpha)\theta\overline{Y}\left[\frac{3}{8}C_x^2\rho_x^* - \frac{1}{2}KC_x^2\sqrt{\rho_y^*\rho_x^*}\right]$ (2.3)

$MSE(T) \approxeq E(T - \overline{Y})^2$

$= \theta\overline{Y}^2\left[C_y^2\rho_y^* + \frac{1}{4}(1 - \alpha)^2C_x^2\rho_x^* - (1 - \alpha)KC_x^2\sqrt{\rho_y^*\rho_x^*}\right]$ (2.4)

To obtain the optimum value of $\alpha$,we have to minimise MSE(T). Differentiating MSE(T) w.r.t. $\alpha$ and equating the derivative to zero; optimum value of $\alpha$ is given by:

$\alpha_{(Opt)} = 1 - 2k\sqrt{\rho_y^*/\rho_x^*}$ (2.5)

Substituting the value of $\alpha_{0pt}$ in (3), we get the minimum value of T as:

$MSE(T)_{Opt} = \theta\overline{Y}^2 C_y^2\rho_y^*[1 - \rho_{yx}^2]$ (2.6)

From equation (2.6) It follows that the proposed estimator T at its optimum condition is equally efficient as that as the usual linear regression estimator.

## 3. Efficiency Comparison:

In systematic sampling, the variance of the usual unbiased estimator $T_0 = \bar{y}_{sys}$ is given by

$$V(T_0) = \theta \bar{Y}^2 C_y^2 \rho_y^* \qquad (3.1)$$

The variance of the ratio estimator $T_1$ suggested by Swain (1964) of population mean $\bar{Y}$ of the study variate based on the systematic sample is given by

$$V(T_1) = \theta \bar{Y}^2 [C_y^2 \rho_y^* + C_x^2 \rho_x^* (1 - 2K\sqrt{\rho^{**}})] \qquad (3.2)$$

The variance of the ratio type exponential estimator $T_2$ suggested by Singh et al. (2011) of population mean $\bar{Y}$ of the study variate based on the systematic sample is given by

$$V(T_2) = \theta \bar{Y}^2 [C_y^2 \rho_y^* + \tfrac{1}{4} C_x^2 \rho_x^* (1 - 4K\sqrt{\rho^{**}})] \qquad (3.3)$$

In this section we compare the optimum MSE of the proposed estimator ($T_{opt}$) with the MSE of simple estimator $T_0$, $T_1$ and $T_2$ and found some theoretical conditions under which the proposed estimator will perform better.

Comparison of $T_0$, $T_1$ and $T_2$ with proposed estimator ($T_{opt}$):

1. $T_{opt}$ is more efficient than simple mean estimator $T_0$ if;

   $$\theta \bar{Y}^2 C_y^2 \rho_y^* \rho_{yx}^2 > 0 \qquad (3.4)$$

2. $T_{opt}$ is more efficient than $T_1$ [suggested by Swain (1964)] if;

   $$\theta \bar{Y}^2 \left[C_x^2 \rho_x^* + C_y^2 \rho_y^* \rho_{yx}^2 - 2KC_x^2 \sqrt{\rho_y^* \rho_x^*}\right] > 0 \qquad (3.5)$$

3. $T_{opt}$ is more efficient than $T_2$ [suggested by Singh et al. (2011)] if;

   $$\theta \bar{Y}^2 \left[\frac{1}{4} C_x^2 \rho_x^* + C_y^2 \rho_y^* \rho_{yx}^2 - KC_x^2 \sqrt{\rho_y^* \rho_x^*}\right] > 0 \qquad (3.6)$$

## 4. Empirical Study:

To illustrate the performance of the proposed modified ratio type estimator (T) relative to the estimator $T_0$, $T_1$ and $T_2$ we assumed (artificially) the following values of the parameters:

$\bar{X}$=44.47, $S_x^2$=149.55, $C_x = 0.28$, $S_{xy} = 538.57$, $\bar{Y} = 80$, $S_y^2 = 2000$, $C_y = 0.56$,

$S_{yz} = -902.86$, $\bar{Z} = 48.40$, $S_z^2 = 427.83$, $C_z$= 0.43, $S_{xz} = -241.06$, $\rho_{xy}$=0.9848,

$P_{yz} = -0.9760$, $\rho_{zx} = -0.9530$, $\rho_x$=0.707, $\rho_y$=0.6652, $\rho_z$=0.5487, N=15, n=3.

**Table 2: MSE of $T_0$, $T_1$, $T_2$ and $T_{opt}$.**

| Estimator | MSE |
|---|---|
| $T_0$ (simple mean estimator) | 1455.08 |
| $T_1$[Swain (1964)] | 373.1425 |
| $T_2$[Singh et al. (2011)] | 820.09 |
| $T_{opt}$ (Proposed) | 44.0705 |

## 5. CONCLUSION

A modified ratio type estimator has been suggested for estimating the finite population mean of the study variable in systematic sampling. The efficiency of the proposed estimator is compared with simple mean estimator as well as the ratio estimator suggested by Swain (1964) and the ratio type exponential estimator suggested by Singh et al. (2011) both theoretically and numerically. Theoretical analysis exhibits that the proposed estimator is more efficient than other estimators under certain conditions.

## REFERENCES

1. Bahl, S. and Tuteja, R.K. (1991). Ratio and Product type Exponential Estimators. Journal of Information and Optimization Sciences. 12, 159-163.
2. Cochran, W.G. (1953). Sampling Techniques. John Wiley and Sons, New-York.
3. Kushwaha, K.S. and Singh, H.P. (1989). Class of almost unbiased ratio and product estimators in systematic sampling. J. Ind. Soc. Agri. Statist, 41(2), 193-205.
4. Murthy, M.N. (1967). Statistical Publishing Society, Calcutta.
5. Singh, H.P. Jatwa, N.K.(2012). A class of exponential type estimators in systematic sampling. Eco. Qulty. Control., 27, 195-208
6. Singh, H.P., Singh, R.(1998). Almost unbiased ratio and product type estimators in systematic sampling. Questiio, 22 (3), 403-416.
7. Singh, H.P., Solanki, R.S. (2012). An efficient class of estimators for the population mean using auxiliary information in systematic sampling. Journal of Statistical Theory and Practice, 6(2), 274-285.
8. Singh, H.P., Tailor, R., Jatwa, N.K., (2011). Modified ratio and product estimators for population mean in systematic sampling. Journals Modern Applied Statistical Methods, 10(2), 424-435.
9. Singh, M. P. (1967) Ratio cum product method of estimation. Metrika, 12,34-42.
10. Shukla, N.D., (1971), Systematic sampling and product method of estimation, in Proceeding of all India Seminar on Demography and Statistics. B.H.U. Varanasi: India.

11. Swain, A.K.P.C. (2014). On an improved ratio type estimator of finite population mean in sample surveys Revista Investi. Opera. 35(1), 49-57.
12. Swain. A.K.P.C.(1964), The use of systematic sampling ratio estimate. Journal of the Indian Statistical Association, 2, 160-164.
13. Tailor, R., N.K. Jatwa and R. Tailor (2014). Ratio-cum-product estimator of population mean in systematic sampling using known parameters of auxiliary variates. Jour. Reliab. Satist. Stud., 7(2).129-138.
14. Tailor, R., Jatwa, N.K., Singh, H.P. (2013) A Ratio-Cum-Product Estimator of Finite Population Mean in Systematic Sampling, Statistics in transition, 14(3), 391-398.